$t\epsilon T$. Let E_N denote the euclidean N-space whose elements are vectors $Y = (y_1, y_2, \ldots, y_N)$. Note that any $t\epsilon T$ can be represented as $t = a_j + y_j\omega_j$ for some 1, $1 \le j \le N$. Clearly,

$$I(t,Y) = \sum_{i=1}^{N} I_i(Y) \qquad \text{where, } I_i(Y) = \int_{-y_i\omega_i}^{a_1 + y_j\omega_j - y_i\omega_i} F_i(\tau)d\tau$$

Let $Y_0\epsilon E_N$. As $I_i(Y)$ is continuous with respect to y_j and y_i separately, for a given $\epsilon' = \epsilon/2N > 0$, there exists a $\delta > 0$ such that $|I_i(Y) - I_i(Y_0)| < \epsilon/N$ and hence, $|I(t(y_j),y) - I(t(y_{0j});y_0)| < \epsilon$ for $||y - y_0|| < \delta$.

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Manuscript received March 2, 1983; revision received August 5, and accepted August 21, 1983.

Hydrodynamic Parameters for Gas-Liquid Cocurrent Flow in Packed Beds

The hydrodynamics of cocurrent gas-liquid flow in packed beds is analyzed by extending the concept of relative permeability to the inertial regime.

The relative permeabilities of the gas and liquid phases are functions of the saturation of the liquid phase. These functions are found from an analysis of experimental data. The relations obtained are used to develop empirical correlations for predicting liquid holdup and pressure drop in gas-liquid cocurrent downflow in packed beds over a wide range of operating conditions. The correlations proposed give very good results when compared to experimental data yielding, in general, mean relative deviations lower than existing correlations. In addition, a new equation is proposed for predicting static holdup in packed beds which is based on a more physically realistic characteristic length than that used in previous studies.

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SCOPE

The prediction of the liquid holdup and pressure drop for two-phase flow in packed beds is of great importance in industrial processes. This kind of problem presents itself in a wide variety of applications, such as in trickle-bed reactors and packed-bed gas-liquid absorbers. The study of flow regimes, pressure drop and holdup in these systems has been conventionally treated from a completely empirical point of view due to the complexity of the problem. This has induced the development of a large diversity of methods of analysis in the literature; as a result, there now exist several different empirical correlations for predicting these hydrodynamic parameters.

The objective of this work is to provide a rational approach to predict pressure drops and liquid holdups for the case of gas-liquid cocurrent down-flow through packed beds and to develop new correlations to estimate those hydrodynamic parameters.

The analysis presented in this work is based on the use of the traditional concepts of capillary pressure and relative permeabilities to model the hydrodynamics of two-phase flow in packed beds. One of the main goals of this study is to determine how the gas and liquid relative permeabilities for both the viscous and inertial flow regimes are affected by the operating conditions, the structure of the medium, and the physical properties of the fluids.

CONCLUSIONS AND SIGNIFICANCE

A semiempirical approach is used to analyze the hydrodynamics of cocurrent gas-liquid flow in packed beds.

A correlation for predicting static holdup is presented (Eq. 8). It is a simple modification of the existent correlation of Charpentier et al. (1968); even though it does not improve considerably the accuracy of the prediction, it is based on a

Work performed while authors were at the Department of Chemical Engineering, University of California, Davis.

more physically realistic characteristic length for the particles. This correlation gives a mean relative deviation of 24.8% when compared to experimental data. The relatively large degree of scatter is due to experimental error and the exclusion from the correlation of other important physical parameters such as the contact angle of the gas-liquid-solid contact line, which describes the wettability of the solid.

The hydrodynamics of gas-liquid flow is studied by extending the concept of relative permeability to the inertial regime. The viscous and inertial liquid relative permeabilities prove to be equal after analyzing 70 experimental points from different investigators. An empirical relationship between the liquid relative permebility and the reduced saturation is found. This allows us to find a correlation for predicting liquid holdup for the specific case of stagnant gas (Eq. 34) which gives a mean relative deviation of 15.2% when compared to 98 experimental data points. The correlation shows a smaller mean relative deviation than that developed by Specchia and Baldi (1977) and is of the same order of accuracy as that developed by Otake and Okada (1953). Most of the data analyzed corresponds to the air-water system.

A larger mean relative deviation for systems with physical properties different from the air-water system suggests that the liquid relative permeabilities may be functions of the Weber number, but there is not enough data at this time to determine this functional dependence.

The viscous and inertial gas phase relative permeabilities are also found to be equal and are correlated to the gas phase saturation. This allows the development of a correlation for predicting liquid holdup for the general gas-liquid flow case in the homogeneous regime (Eq. 51) as well as a correlation for predicting pressure drops (Eq. 50).

The prediction of liquid holdup gives a mean relative deviation of 12.7% when compared to 59 experimental points which proves to be much better than the results obtained by applying the correlations of Midoux et al. (1976) and Clements and Schmidt (1980).

The prediction of pressure drops gives a mean relative deviation of 21.9% in the analysis of 49 experimental points, improving the predictions of the correlations developed by Larkins (1959) and Midoux et al. (1976).

Further analysis is needed to determine the effect of additional parameters, such as the Weber number, on the relative permeability relations.

The most important advantage of the approach presented in this work is that the pressure drop and liquid holdup correlations are developed by determining empirically the relations between relative permeabilities and saturations only, whereas the conventional treatment of the problem is based on the determination of several adjustable parameters which are specific for the variable that is being correlated. Our approach establishes a direct relationship between pressure drop and liquid holdup since both of these hydrodynamic parameters are ultimately related to the relative permeabilities.

INTRODUCTION

Extensive literature reviews are available on the hydrodynamics of cocurrent gas-liquid flow in packed beds (Gianetto et al., 1978; Satterfield, 1975; Shah, 1979; Sáez, 1984). Much of the effort devoted to this subject has been directed toward the prediction of hydrodynamic parameters, given the operating conditions and the properties of the packing materials and fluid phases. Special emphasis has been given to the study of flow regimes, pressure drops, and liquid-phase saturations, parameters that are of great importance in the design of trickle-bed reactors and packed-bed gas-liquid absorbers.

Flow Regimes

The presence of different hydrodynamic regimes in cocurrent gas-liquid flow in packed beds was first recognized in the early experiments of McIlvreid (1956) and Larkins (1959). Since that time, many investigators have studied the transition between the various types of flow. It is widely accepted that four general regimes of flow occur in these processes: the homogeneous or low interaction regime, in which both phases are continuous, the spray flow regime, the dispersed bubble flow regime, and the pulsing flow regime, in which the liquid phase accumulates in well-defined horizontal regions in the bed and travels through the medium in the form of pulses or slugs. This flow regime is often called the high interaction regime.

Weekman (1963) studied the influence of gas and liquid flow rates on the type of flow regime, whereas Charpentier et al. (1969), Beimesch (1972), and Charpentier and Favier (1975) concentrated on the effect of fluid physical properties on the transition between these types of flow. Chou et al. (1977) and Gianetto et al. (1970, 1978) observed that the packing characteristics and the structure of the medium had a definite influence on the occurrence of the different flow regimes. Several investigators (Charpentier and Favier, 1975; Chou et al., 1977; Talmor, 1977; Specchia and Baldi, 1977) have proposed flow maps for both foaming and nonfoaming systems.

These studies suggest empirical relations to determine the type of flow regime as a function of the operating characteristics of the system. Considering that in industrial applications of trickle-bed reactors and gas-liquid absorbers one can encounter wide ranges of operating conditions, fluid physical properties, and types of packing (Satterfield, 1975; Hofmann, 1977), there is a strong need to have a way of predicting flow regimes more reliably. In this work we are not developing a predictive method for determining flow regimes; we will be analyzing experimental holdup and pressure drop data only in the low interaction regime.

Pressure Drop

The two-phase pressure drop through a packed bed is perhaps the most important hydrodynamic parameter from the point of view of system design. Most of the correlations developed to predict pressure drops for two-phase flow are based on an initial estimate of single-phase pressure drops through the bed. There is a considerable amount of work on the calculation of single-phase pressure drops but the most widely used correlation is that proposed by Ergun (1952). Ergun used the definition of the friction factor for laminar and turbulent flow in a capillary tube and applied it to packed beds by defining an equivalent hydraulic radius for the pore space between particles.

The applicability of the Ergun equation has been widely studied. Handley and Heggs (1968) noticed that the coefficients in the equation were not universal constants as Ergun had claimed. Macdonald et al. (1979) proposed a modified form of the Ergun equation and reported that this new correlation is reliable to an accuracy of 50% only. Part of the reason for this scatter is the apparent effect of particle roughness on the pressure drop.

Larkins (1959) was one of the first investigators to propose a correlation for predicting pressure drops in two-phase cocurrent downflow in packed beds. His approach was based on considerations taken from the study of two-phase flow in empty pipes performed by Lockhart and Martinelli (1949). The same kind of approach was followed by subsequent workers such as Weekman (1963), Reiss (1967), Sato and Hashiguchi (1973), and Midoux et al. (1976).

Turpin and Huntington (1967) used a different approach based on the development of a correlation for the friction factor as a function of the operating conditions. This approach was also followed by Specchia and Baldi (1977) to correlate pressure drops in the high interaction regime.

Other two-phase flow pressure drop correlations are based on Ergun-type equations and models involving two-phase flows in capillary tubes and inclined open channels. The latter approach was used by Hutton and Leung (1974) and, more recently, in a two-parameter model proposed by Specchia and Baldi (1977) for the low interaction regime.

Liquid Holdup

The retention of liquid in the bed at certain operating conditions is usually interptreted as resulting from two different contributions: 1. a specific amount of liquid that remains in the bed after it has been drained, called the static or residual holdup; and 2. the difference between the actual amount of liquid in the bed and the residual holdup, known as the dynamic holdup.

The static holdup results from a balance between surface tension forces and gravitational effects. Dombrowski and Brownell (1954) and Nenninger and Storrow (1958) developed correlations to predict the residual holdup considering these effects. More recently, Charpentier et al. (1968) related the static holdup to the Eötvös number, which represents the ratio of gravitational to surface tension forces.

There are several approaches in the literature to correlating the dynamic holdup with the operating conditions. One of these, developed by Otake and Okada (1953) and subsequently applied by several investigators such as Specchia and Baldi (1977), consists in correlating the dynamic holdup to the Reynolds and Galileo numbers and properties of the packing. This approach is generally applied to the special case in which the gas is stagnant in the bed.

Another approach is based on an analogy with two-phase flow in pipes. This method was originally proposed by Larkins (1959) and it has been used subsequently by Midoux et al. (1976) and other investigators. In these correlations the dynamic holdup is directly related to the pressure drops.

More recently, Clements and Schmidt (1980) correlated the dynamic holdup to a dimensionless group that contains the Reynolds numbers of both phases and the Weber number of the gas phase. This has been one of the first attempts to include surface tension effects in the prediction of liquid holdup.

Despite the large amount of work done in this area, there have been very few comparisons of the models and correlations proposed. As a result, the choice of the appropriate correlation to be used for a given system has to be based on existing data taken under operating conditions that resemble as closely as possible what is perceived to be the desired design. This is a very difficult situation in most cases since industrial operations are usually far from the range of operating conditions used in laboratory experiments. Furthermore, the predicted pressure drops and liquid holdups obtained from various correlations can vary over one or two orders of magnitude, and an a priori choice is nearly impossible.

In the following sections we will use the concept of relative permeabilities to propose correlations to evaluate pressure drops and liquid holdup for two-phase downflow in packed beds. The relative permeabilities will be empirically correlated to the reduced saturations of the liquid and gas phases. Before we proceed with the analysis of the two-phase flow problem, we consider the static case and the prediction of the static holdup. This quantity is of importance in determining the mobile liquid-phase volume fraction in the subsequent analysis.

STATIC HOLDUP

A dimensional analysis of the point equations describing the hydrostatics in a porous medium made up of nonporous particles (see Sáez, 1984) leads to the conclusion that the static holdup, ϵ_β^o, is a function of the Eötvös number, the contact angle at the gas-liquid-solid contact line, and the geometry of the packing.

$$\epsilon_\beta^o = \epsilon_\beta^o(E\ddot{o}_l, \theta_c, \text{geometry of particles}) \tag{1}$$

The Eötvös number, $E\ddot{o}_l$, is defined by

$$E\ddot{o}_l = \frac{\rho_\beta g l^2}{\sigma} \tag{2}$$

where l is a characteristic length associated with the intraparticle spaces.

If we consider perfectly wettable solids, we have $\theta_c = 0$ (constant). One might excect that the dependence of ϵ_β^o on geometry is very strong. The best approach at this point seems to be to choose a characteristic length l that minimizes the dependence of ϵ_β^o on the geometry of the particle surface.

Charpentier et al. (1968) proposed that l be chosen as the nominal diameter

$$l = d_p \tag{3}$$

so that their definition of the Eötvös number is

$$E\ddot{o} = \frac{\rho_\beta g d_p^2}{\sigma} \tag{4}$$

There is a disadvantage to this choice of characteristic length. For example, this approach would predict the same ϵ_β^o for a bed packed with solid cylinders and a bed packed with hollow cylinders (Raschig rings) of the same external dimensions. Furthermore, it would predict the static holdup to be completely independent of the porosity of the porous medium.

The characteristic length that best represents hydraulic phenomena in a porous medium is the hydraulic radius, or ratio of void volume to particle surface area. We can choose l to be six times that ratio, i.e.,

$$l = \frac{d_e \epsilon}{1 - \epsilon} \tag{5}$$

where d_e is the equivalent particle diameter, defined by

$$d_e = \frac{6V_p}{S_p} \tag{6}$$

Using this characteristic length in the definition of the Eötvos number, we obtain

$$E\ddot{o}^* = \frac{\rho_\beta g d_e^2 \epsilon^2}{\sigma(1 - \epsilon)^2} \tag{7}$$

Experimental data of static holdup were collected from the literature for different packing materials and various fluids. The sources are presented in Table 1.

Figure 1 is a plot of ϵ_β^o vs. $E\ddot{o}^*$. The solid line corresponds to a proposed fit, given by the equation

$$\epsilon_\beta^o = \frac{1}{20 + 0.9E\ddot{o}^*} \tag{8}$$

TABLE 1. EXPERIMENTAL DATA: STATIC HOLDUP

References	System	Packing	Symbol
Otake and Okada (1953)	Air-Water	Raschig Rings	○
	Air-Water	Berl Saddles	●
Charpentier et al. (1968)	Air-Water	Raschig Rings	○
	Air-80% Isopropanol Solution	Raschig Rings	○
	Air-70% Sucrose Solution	Raschig Rings	○
Specchia and Baldi (1977)	Air-Water	Spheres	△
	Air-Water	Cylinders	▲
Beimesch (1972)	Air-Water	Raschig Rings	○
	Air-Water	Checker Marbles	□
Carley and Laswell (1980)	Air-Shale Oil	Crushed Oil Shale	■

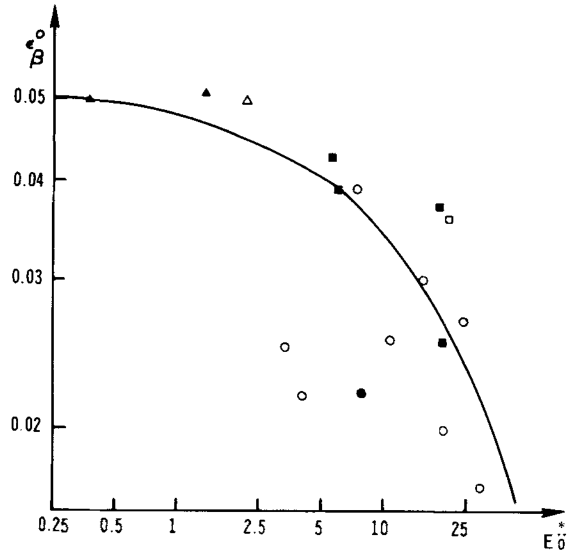

Figure 1. Static holdup correlation.

There is no question that the data in Figure 1 are widely scattered and that Eq. 8 is only an approximate representation of the physical observations. There are two major contributions for the failure of the static holdup data to be correlated very well by the suggested functional dependence. First, the experimental measurement of the static holdup involves a fair amount of error since the amounts of liquid to be measured are small and artifacts such as liquid retention in other parts of the column and accessory equipment can play an important role. Second, we are not taking into account the wettability of the solid in the correlation. We are assuming that the solid is perfectly wetted by the liquid. When this assumption is not valid, the result is a value of ϵ_β^o below that corresponding to perfect wetting and we see that the correlation tends to overestimate the experimental data on the average.

One should also point out that Eq. 8 predicts an asymptotic value of ϵ_β^o for small Eötvös numbers. The limiting value of $\epsilon_\beta^o = 0.05$ is supported by more experimental evidence than has been shown here and Charpentier et al. (1968) have summarized the relevant sources of this limit for static holdup.

The mean relative deviation of the proposed correlation when compared to 17 experimental points is 25%. The representation of Charpentier et al. (1968) gives a deviation of 27% for the same data points.

The conclusion is that either of the two representations can be used as a way of estimating ϵ_β^o from a practical standpoint. However, the new representation avoids some possible pitfalls in the characteristic length chosen by previous investigators.

In any case, there is a need for more research in this area, especially in the theoretical prediction of static holdup and in the influence of the degree of wettability of the solid phase.

DYNAMIC HOLDUP AND PRESSURE DROP PREDICTION

The macroscopic equations of motion governing two-phase flow in packed beds are discussed in detail by Sáez (1984). They are derived by applying the volume averaging technique (Whitaker, 1969; Carbonell and Whitaker, 1982). The final form of the equations, written for the α-phase (where α can be either liquid or gas) is

$$\epsilon_\alpha \rho_\alpha \langle v_\alpha \rangle^\alpha \cdot \nabla \langle v_\alpha \rangle^\alpha = -\epsilon_\alpha \nabla \langle P_\alpha \rangle^\alpha - F_\alpha + \mu_\alpha \nabla^2 \langle v_\alpha \rangle^\alpha \tag{9}$$

This equation corresponds to the case of steady, incompressible flow of Newtonian fluids. Furthermore, we are considering that the α-phase is continuous in the porous medium.

In Eq. 9, F_α represents a force exerted by the α-phase on the other two phases present in the medium. It is made up of the integration of viscous and pressure forces at the interfaces.

The macroscopic viscous term, $\mu_\alpha \nabla^2 \langle v_\alpha \rangle^\alpha$, often called the Brinkmann correction, is not very significant as pointed out by Slattery (1978) except perhaps in the vicinity of walls surrounding a porous medium. Its effect quickly diminishes a distance of one or two particles away from the wall.

Choudhary et al. (1976) studied the importance of the macroscopic inertial terms for the case of one-phase flow through a packed bed with strong packing nonuniformities and found that these terms were negligible even at high Reynolds numbers.

Following these arguments, we can neglect the macroscopic viscous and inertial contributions in Eq. 9 to get

$$\nabla \langle P_\alpha \rangle^\alpha = -\frac{F_\alpha}{\epsilon_\alpha} \tag{10}$$

At this point, a constitutive equation for the force F_α is needed. For the case of one-phase flow through a porous medium, the force F_α can be expressed in terms of the Ergun equation as follows

$$\frac{F_\alpha^o}{\epsilon_\alpha} = \left\{ A \frac{Re_\alpha^*}{Ga_\alpha^*} + B \frac{Re_\alpha^{*2}}{Ga_\alpha^*} \right\} \rho_\alpha g \hat{u}_\alpha \tag{11}$$

where $\hat{u}_\alpha$ is a unit vector in the direction of the average velocity, $\langle v_\alpha \rangle^\alpha$. The superscript o indicates one-phase flow of phase α through the medium.

We can see that the force is made up of two different contributions: the viscous term, $A\ Re_\alpha^*/Ga_\alpha^*$, and the inertial term, $B\ Re_\alpha^{*2}/Ga_\alpha^*$. These contributions are independent of each other and they dominate at low and high values of the Reynolds number, respectively. For the general case of two-phase flow, we propose a similar constitutive equation for the force F_α in which each contribution to the force corresponding to one-phase flow is scaled by an unknown factor, as follows:

$$\frac{F_\alpha}{\epsilon_\alpha} = \left\{ \frac{A}{k_\alpha} \frac{Re_\alpha^*}{Ga_\alpha^*} + \frac{B}{k_{\alpha i}} \frac{Re_\alpha^{*2}}{Ga_\alpha^*} \right\} \rho_\alpha g \hat{u}_\alpha \tag{12}$$

The quantities k_α and $k_{\alpha i}$ are defined as the viscous and inertial relative permeabilities, respectively.

The concept of viscous relative permeability has been widely applied to the problem of multiphase flow through porous media (Scheidegger, 1974) and it is usually assumed that k_α is only a function of the saturation of the α phase, S_α. The application of the concept of inertial relative permeability has been used less frequently, mostly because the traditional studies of multiphase flow in porous media deal with low Reynolds number cases.

Let us consider now the case of two-phase downflow through a packed bed. We will consider that the flow is one-dimensional and that the volume fractions of liquid (ϵ_β) and gas (ϵ_γ) are uniform with respect to the horizontal coordinates x,y. Under these conditions we can combine Eqs. 10 and 12 for each phase to obtain

$$-\frac{d\langle P_\beta \rangle^\beta}{dz} + \rho_\beta g = \left\{ \frac{A}{k_\beta} \frac{Re_\beta^*}{Ga_\beta^*} + \frac{B\ Re_\beta^{**}}{k_{\beta i}\ Ga_\beta^*} \right\} \rho_\beta g \tag{13}$$

$$-\frac{d\langle P_\gamma \rangle^\gamma}{dz} + \rho_\gamma g = \left\{ \frac{A}{k_\gamma} \frac{Re_\gamma^*}{Ga_\gamma^*} + \frac{B}{k_{\gamma i}} \frac{Re_\gamma^{*2}}{Ga_\gamma^*} \right\} \rho_\gamma g \tag{14}$$

The continuity equations, expressed in terms of the superficial mass fluxes of gas and liquid, G and L, are

$$\rho_\beta \epsilon_\beta \langle v_\beta \rangle^\beta = L \tag{15}$$

$$\rho_\gamma \epsilon_\gamma \langle v_\gamma \rangle^\gamma = G \tag{16}$$

We also know that the volume fraction of both phases must add up to the total fraction of voids in the medium, ϵ.

$$\epsilon_\beta + \epsilon_\gamma = \epsilon \tag{17}$$

Furthermore, we can introduce the definition of capillary pressure, P_c,

$$P_c = \langle P_\gamma \rangle^\gamma - \langle P_\beta \rangle^\beta \tag{18}$$

Equations 13 to 18 define the problem completely in terms of the six unknowns $\langle P_\beta \rangle^\beta$, $\langle P_\gamma \rangle^\gamma$, $\langle v_\beta \rangle^\beta$, $\langle v_\gamma \rangle^\gamma$, ϵ_β, ϵ_γ, providing that the capillary pressure and the relative permeabilities are known functions of the operating conditions.

We can subtract Eq. 14 from Eq. 13 and make use of Eq. 18 to get

$$\frac{dP_c}{dz} + (\rho_\beta - \rho_\gamma)g = -\left\{\frac{180}{k_\gamma}\frac{Re_\gamma^*}{Ga_\gamma^*} + \frac{1.8}{k_{\gamma i}}\frac{Re_\gamma^{*2}}{Ga_\gamma^*}\right\}\rho_\gamma g + \left\{\frac{180}{k_\beta}\frac{Re_\beta^*}{Ga_\beta^*} + \frac{1.8}{k_{\beta i}}\frac{Re_\beta^{*2}}{Ga_\beta^*}\right\}\rho_\beta g \quad (19)$$

In this equation, we have taken the Ergun constants A and B equal to 180 and 1.8, respectively, as recommended by Macdonald et al. (1979).

For the case of a nearly static fluid, the capillary pressure P_c can be related to the dimensionless Leverett J function by (Scheidegger, 1974)

$$P_c = \sigma\left(\frac{\epsilon}{K}\right)^{1/2} J(S_\beta) \quad (20)$$

The correlation J vs. S_β is not unique and appears to be dependent on the structure of the porous medium, as pointed out by Bear (1972). The Leverett function also shows hysteresis in that there is an appreciable difference between the values of J at a fixed saturation in an imbibition experiment from the values of J measured upon drainage of the porous medium. We are going to consider that ϵ_β does not change appreciably with z, which implies that dP_c/dz is negligible in Eq. 19, providing that ϵ and k are uniform throughout the medium and Eq. 20 is applicable. This hypothesis is always made in experimental and theoretical studies found in the literature. However, we are not aware of a rigorous test of this assumption.

Under these conditions, Eq. 19 becomes

$$\left\{\frac{180}{k_\beta}\frac{Re_\beta^*}{Ga_\beta^*} + \frac{1.8}{k_{\beta i}}\frac{Re_\beta^{*2}}{Ga_\beta^*}\right\}\frac{\rho_\beta}{\rho_\beta - \rho_\gamma} - \left\{\frac{180}{k_\gamma}\frac{Re_\gamma^*}{Ga_\gamma^*} + \frac{1.8}{k_{\gamma i}}\frac{Re_\gamma^{*2}}{Ga_\gamma^*}\right\}\frac{\rho_\gamma}{\rho_\beta - \rho_\gamma} = 1 \quad (21)$$

If the relative permeabilities were known functions of the operating conditions and holdup, Eq. 21 would be a relation that could be used to evaluate the holdup given the operating conditions. Our next goal is to develop empirical equations for the relative permeabilities. First, we will consider the case of a stagnant gas phase with liquid downflow through a packed bed.

Liquid Holdup—Stagnant Gas

Consider that the gas is stagnant in the bed, i.e.,

$$\langle v_\gamma \rangle = 0 \quad (22)$$

Then Eq. 21 becomes (considering $\rho_\beta \gg \rho_\gamma$)

$$\frac{Re_\beta^*}{Ga_\beta^*}\left(\frac{180}{k_\beta} + \frac{1.8\,Re_\beta^*}{k_{\beta i}}\right) = 1 \quad (23)$$

At this point we will adopt the hypothesis that the liquid relative permeabilities are only functions of δ_β, the reduced saturation (Sáez, 1984)

$$\delta_\beta = \frac{\epsilon_\beta - \epsilon_\beta^o}{\epsilon - \epsilon_\beta^o} \quad (24)$$

$$k_\beta = k_\beta(\delta_\beta) \quad (25)$$

$$k_{\beta i} = k_{\beta i}(\delta_\beta) \quad (26)$$

The reduced saturation represents the ratio of the effective volume of flow of the liquid phase to the available volume of flow, considering that the static holdup, ϵ_β^o, represents a portion of the void fraction occupied by stagnant liquid. Hence, Eqs. 25 and 26 must satisfy the limits

$$\lim_{\delta_\beta \to 0} k_\beta = \lim_{\delta_\beta \to 0} k_{\beta i} = 0 \quad (27)$$

$$\lim_{\delta_\beta \to 1} k_\beta = \lim_{\delta_\beta \to 1} k_{\beta i} = 1 \quad (28)$$

TABLE 2. EXPERIMENTAL DATA: LIQUID HOLDUP—STAGNANT GAS

References	System	Packing	Symbol
Uchida and Fujita (1938)	Air-Water	Raschig Rings	●
Otake and Okada (1953)	Air-Water	Raschig Rings	○
Charpentier and Favier (1975)	Air-Water, Air-Cyclohexane	Spheres	■
		Cylinders	□
Specchia and Baldi (1977)	Air-Water	Spheres	▲
		Cylinders	△
Charpentier et al. (1968, 1969)	Air-Water, Isopropanol, and Sucrose Sol'ns	Raschig Rings	▼
Wijffels et al. (1974)	Air-Water	Spheres	▽
Jesser and Elgin (1943)	Air-Water, Tergitol no. 7 Sol'ns	Spheres	◇
		Berl Saddles	◆
		Carbon Rings	X

The next step is to find the functional forms of Eqs. 25 and 26 from experimental data available in the literature. For this purpose, we have collected experimental data from several investigators. The data selected are presented in detail by Sáez (1984) and the sources are listed in Table 2. These data include several kinds of packing as well as a wide range of experimental conditions. The liquid flow rate per unit area of column ranges from 0.153 to 31.5 $kg/m^2\cdot s$. Most of the data points analyzed correspond to the air-water system. Other liquids used were isopropanol solutions, sucrose solutions, and cyclohexane.

Now we propose functional relationships for the relative permeabilities as follows

$$k_\beta = \delta_\beta^a, \quad a > 0 \quad (29)$$

$$k_{\beta i} = \delta_\beta^b, \quad b > 0 \quad (30)$$

Note that Eqs. 29 and 30 agree with the limits stated by Eqs. 27 and 28. The values of a and b can be calculated by minimizing the relative deviation of the value of δ_β calculated by combining Eqs. 29 and 30 with Eq. 23 with respect to the experimental value $\delta_{\beta,\mathrm{EXP}}$.

The problem can be stated as

$$\text{Minimize } \hat{e}(a,b) = \frac{1}{N}\sum_{i=1}^{N} |\delta_{\beta i,\mathrm{CALC}} - \delta_{\beta i,\mathrm{EXP}}| \times \frac{100}{\delta_{\beta i,\mathrm{EXP}}}$$

where N is the number of data points analyzed. The optimization procedure was carried out by using a direct search method based on the "regula falsi" technique. The process was performed for the particular case $a = b$ and for the general case. The optimum values for a and b are shown in Table 3. Other kinds of fitting functions were used instead of Eqs. 29 and 30, such as polynomial and exponential functions, but the errors were higher for those cases. The results indicate that we can take $a = b$ as a definitive result since the improvement of taking the two-parameter case is only 3.4%, which is certainly within the range of experimental error.

The final results can be expressed as

$$k_{\beta i} = k_\beta \quad (31)$$

$$k_\beta = \delta_\beta^{2.43} \quad (32)$$

Figures 2 and 3 show how Eq. 32 compares with experimental values. The experimental values of k_β are found by combining Eqs. 23 and 31 to get

$$k_\beta = 180\frac{Re_\beta^*}{Ga_\beta^*} + 1.8\frac{Re_\beta^{*2}}{Ga_\beta^*} \quad (33)$$

TABLE 3. OPTIMUM VALUES OF a AND b: LIQUID RELATIVE PERMEABILITIES

Case	a	b	$\hat{e}(\%)$
$a = b$	2.43	2.43	14.8
$a \neq b$	2.2	2.7	11.4

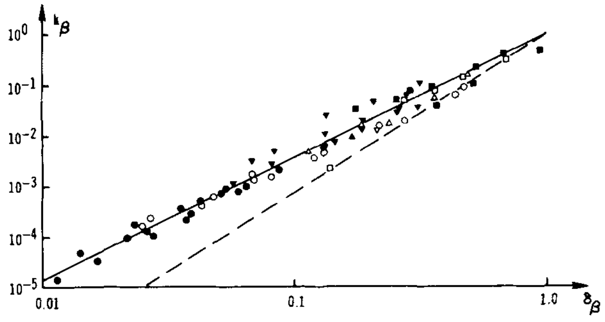

Figure 2. Liquid relative permeability: logarithmic scale.

The solid lines in Figures 2 and 3 correspond to Eq. 32. The dotted lines in those figures correspond to the prediction of k_β given by a one-dimensional capillary tube model developed by Sáez (1984). Note that, even though there is appreciable scatter of the data, Eq. 32 is a good representation of the experimental points. The capillary tube model underestimates the value of k_β. This is due in part to the fact that the static holdup is not considered in that model. Note that, as a result, the difference between the dotted line and the experimental points becomes more appreciable as δ_β goes to zero. However, the simple capillary tube model provides a correct idea of the trend for the relative permeability curve even though it does not explicitly consider turbulence effects.

Equations 32 and 33 allow us to propose a correlation for predicting liquid holdup in packed beds for the case of stagnant gas phase. This correlation is given by

$$\delta_\beta = \left(180\frac{Re_\beta^*}{Ga_\beta^*} + 1.8\frac{Re_\beta^{*2}}{Ga_\beta^*}\right)^{0.41} \tag{34}$$

Figure 4 compares the values of $\delta_\beta Ga_\beta^{*0.41}$ calculated from Eq. 34 with the experimental data.

Equation 34 can be compared to the correlations available in the literature. Specifically, we will compare the results of applying this equation with the correlations developed by Otake and Okada (1953) and Specchia and Baldi (1977).

The mean relative deviations for the three correlations are presented in Table 4. The correlation by Otake and Okada gives an agreement that is, on the average, as good as the one provided by the correlation developed in this work. The relation suggested by Specchia and Baldi gives a mean deviation higher than the 22% reported in the original work for the mean quadratic deviation.

These results indicate that the correlation developed in this work can be used with confidence to predict liquid holdup in packed beds for the case of no gas flow.

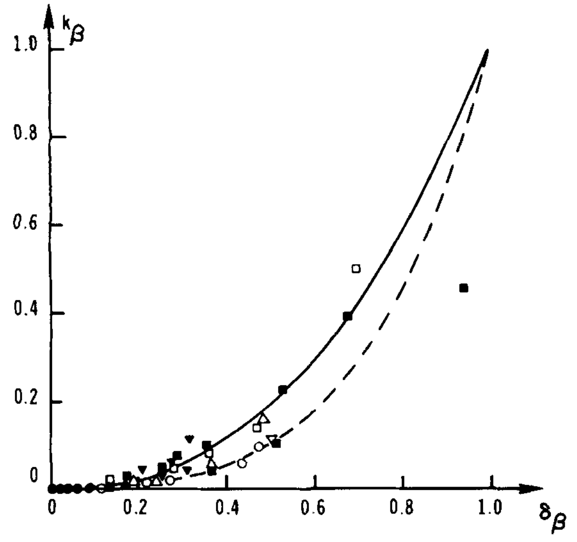

Figure 3. Liquid relative permeability: linear scale.

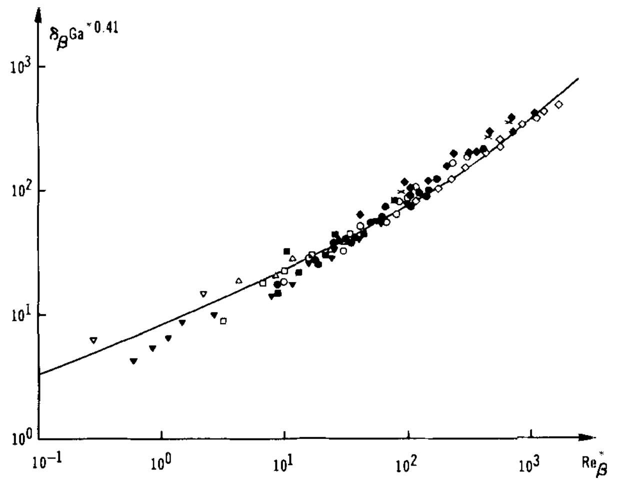

Figure 4. Correlation for predicting liquid holdup: stagnant gas phase.

It is interesting to point out that the correlations developed by Otake and Okada and Specchia and Baldi contained four parameters to be fitted with experimental data and the analysis performed in this work only required the evaluation of one parameter (the exponent of δ_β in the equation for liquid relative permeability) plus the confirmation of the equality of k_β and $k_{\beta t}$.

An analysis of the data shows that there is an apparent effect of the physical properties of the fluids on the dependence of k_β on δ_β. The prediction of δ_β gives a mean relative deviation of 11.9% in 57 experimental points for the air-water system, whereas the mean relative deviation is 28.3% in 13 data points corresponding to other fluids. This result suggests that k_β is also a function of the physical properties of the fluids. This dependence must be based on the effect of the Reynolds, Galileo and Weber numbers, which are the basic dimensionless parameters of the problem.

All the data analyzed so far were used to fit k_β as a function of δ_β. The next step is to apply the correlation developed to independent data and see if the deviations obtained are in the same range as for the previous analysis.

For this purpose, we analyzed the data reported by Jesser and Elgin (1943). A favorable characteristic of these data is the wide variety of packing structures investigated.

The results show a mean relative deviation of 16% for 28 points, which compares very well to the deviation of 14.9% for the data reported in the references listed in Table 2. This is very encouraging, especially since the data of Jesser and Elgin are at higher Reynolds numbers than those analyzed previously. The relative deviation of all the data analyzed (98 points) using Eq. 34 is 15.2% which is a very acceptable result.

Jesser and Elgin (1943) isolated the effect of surface tension on the liquid holdup by performing experiments in which σ was altered by adding a surfactant (Tergitol No. 7) to water. The surfactant did not alter the values of viscosity and density. The experiments were performed using carbon rings as packing particles and two solutions of Tergitol No. 7 with surface tensions of: (1) 37.3 $\times 10^{-3}$ N/m and (2) 28.2 $\times 10^{-3}$ N/m. We will compare the predictions of our model with these experimental data.

We base our comparison on the ratio $\epsilon_{\beta,i}^D/\epsilon_{\beta,o}^D$ where the subscript o refers to pure water and the subscript i denotes the number of the Tergitol solution at the same Reynolds and Galileo numbers.

By comparing Eqs. 8, 24, and 34, we can show that

TABLE 4. MEAN RELATIVE DEVIATIONS: REDUCED LIQUID SATURATION

Correlation	$\hat{e}(\%)$
Otake and Okada (1953)	14.3
Specchia and Baldi (1977)	31.8
This Work	14.9

TABLE 5. SURFACE TENSION EFFECTS ON DYNAMIC HOLDUP

$L(kg/m^2 \cdot s) \times 10^2$	$\epsilon^D_{\beta,1}/\epsilon^D_{\beta,0}$ Exp.	Calc.	$\epsilon^D_{\beta,2}/\epsilon^D_{\beta,0}$ Exp.	Calc.
542.5	1.26	1.81	1.48	2.12
1,356.3	1.24	1.81	1.32	2.12
2,712.5	1.13	1.81	1.17	2.12
4,068.8	1.07	1.81	1.10	2.12

$$\frac{\epsilon^D_{\beta,i}}{\epsilon^D_{\beta,o}} = \frac{\zeta_A}{\zeta_B} \cdot \frac{\zeta_B \epsilon - 1}{\zeta_A \epsilon - 1} \tag{35}$$

where

$$\zeta_A = 0.9 + \frac{20}{E\ddot{o}^*_o} \tag{36}$$

$$\zeta_B = 0.9 \frac{\sigma_o}{\sigma_i} + \frac{20}{E\ddot{o}^*_o} \tag{37}$$

and $E\ddot{o}^*_o$ is the Eötvös number calculated using σ_o.

It is interesting to point out that the effect of σ on the dynamic holdup according to our model comes in only through the value of ϵ^o_β.

Table 5 shows the experimental and calculated values of $\epsilon^D_{\beta,i}/\epsilon^D_{\beta,o}$. From these results we can see that the ratio of dynamic holdups tends to be overestimated by our correlation. This could be due in part to the lack of accuracy of Eq. 8. However, the most interesting observation is that the effect of surface tension is a function of L, the liquid mass flux. This suggests that the relative permeability is affected by the Weber number of the flow. This is quite logical since k_β is directly related to the shape of the liquid-gas interface and it, in turn, is a function of surface forces.

This analysis indicates that further studies have to be performed in order to find out the dependence of the relation between k_β and δ_β on the basic dimensionless groups (Re, We, Ga). It is important to point out that the effect of packing structure seems to be well defined in the variable δ_β, since the dependence of k_β on δ_β is not strongly affected by the size, shape, or porosity of the packing.

Pressure Drop and Liquid Holdup—Two-Phase Flow

Consider the general case of cocurrent gas-liquid downflow in packed beds. The pressure drop in the gas phase is given by Eq. 14. If we consider (as before) that changes in liquid holdup are negligible in the z direction, Eq. 14 can be integrated to obtain

$$-\frac{\Delta \langle \mathcal{P}_\gamma \rangle^\gamma}{\mathcal{L}} = \left\{ \frac{A}{k_\gamma} \frac{Re^*_\gamma}{Ga^*_\gamma} + \frac{B}{k_{\gamma i}} \frac{Re^{*2}_\gamma}{Ga^*_\gamma} \right\} \rho_\gamma g \tag{38}$$

where

$$\langle \mathcal{P}_\gamma \rangle^\gamma = \langle P_\gamma \rangle^\gamma - \rho_\gamma g \mathcal{L} \tag{39}$$

and $\mathcal{L}$ is the distance over which the pressure drop is measured. The gravitational term in Eq. 39 is usually negligible for the gas phase so that $\langle \mathcal{P}_\gamma \rangle^\gamma$ and $\langle P_\gamma \rangle^\gamma$ are equivalent. In any case, $\Delta \langle \mathcal{P}_\gamma \rangle^\gamma$ is what is measured experimentally.

Let us define a dimensionless pressure drop per unit length

$$\psi_\gamma = \frac{1}{\rho_\gamma g} \left(-\frac{\Delta \langle \mathcal{P}_\gamma \rangle^\gamma}{\mathcal{L}} \right) \tag{40}$$

So that Eq. 38 becomes

$$\psi_\gamma = \frac{A}{k_\gamma} \frac{Re^*_\gamma}{Ga^*_\gamma} + \frac{B}{k_{\gamma i}} \frac{Re^{*2}_\gamma}{Ga^*_\gamma} \tag{41}$$

We will now rely upon the hypothesis stated previously that the relative permeabilities are only functions of the reduced saturations. If we assume that the irreducible saturation of the gas phase is negligible then $\delta_\gamma = S_\gamma$ and we can state

$$k_\gamma = k_\gamma(S_\gamma) \tag{42}$$

$$k_{\gamma i} = k_{\gamma i}(S_\gamma) \tag{43}$$

TABLE 6. EXPERIMENTAL DATA FOR AIR-WATER SYSTEM: LIQUID HOLDUP AND PRESSURE DROP—TWO-PHASE FLOW

Reference	Packing	d_c/d_p	Symbol
Beimesch (1972)	Raschig Rings	24	△
	Checker Marbles	12	◇
Charpentier et al. (1968), 1969)	Raschig Rings	9, 10, 16	□
Larkins (1959)	Cylinders	32	■
	Raschig Rings	10	▽
	Spheres	11	▼
Matsuura and Akehata (1979)	Spheres	31	○
Specchia and Baldi (1977)	Spheres	14	▲
	Cylinders	15	●

The next step is to find the functional relationships (Eqs. 42 and 43) from experimental data on pressure drops for two-phase flow.

There is a large amount of experimental data available in the literature concerning pressure drop and liquid holdup for cocurrent gas-liquid downflow. A partial list is shown in Table 6. However, a large portion of these data exhibit one or more of the following characteristics.

1. Only one of the two key parameters (pressure drop or liquid holdup) is reported.
2. The ratio of column to particle diameter is small. Cohen and Metzner (1981) have shown that, when this ratio is smaller than 20, velocity nonuniformities due to porosity variations near the wall of the bed can be very significant.
3. The data for one-phase flow do not follow the Ergun equation with the usual coefficients, even though the ratio of column diameter to particle diameter is large.

Points 1 and 2 are determining factors used in this work to screen the data used to correlate k_γ with S_γ. We only used data with a ratio of column to particle diameter greater than 20 for which the effect of velocity nonuniformities is usually less than 10% (see Cohen and Metzner, 1981), and for which both pressure drop and holdups are reported.

It is interesting to note that most of the data for one-phase flow did not agree well with the coefficients suggested by Macdonald et al. (1979) for the Ergun equation ($A = 180$, $B = 1.8$). The reason for this could not be determined from the information available. One of the possibilities for this behavior of the data could be, as Macdonald et al. (1979) suggest, the effect of the roughness of the particles. Another possibility is that of channeling due to improper bed packing.

Following the approach used to correlate the liquid relative permeabilities, the gas relative permeabilities k_γ and $k_{\gamma i}$ were fitted to equations of the form

$$k_\gamma = S^c_\gamma \tag{44}$$

$$k_{\gamma i} = S^d_\gamma \tag{45}$$

The experimental data for liquid holdup were used to calculate S_γ in Eqs. 44 and 45. The experimental pressure drop data were used to compute ψ_γ in Eq. 41. Using the same optimization procedure that was used to correlate k_β, the following optimum values of c and d were obtained

$$c = d = 4.8 \tag{46}$$

The final equations for the gas-phase relative permeabilities are then given by

$$k_{\gamma i} = k_\gamma \tag{47}$$

and

$$k_\gamma = S^{4.80}_\gamma \tag{48}$$

Again these relations were developed from experimental data listed in Table 6 for which the ratio of column to particle diameter was greater than 20, and for which both pressure drops and liquid holdups were reported.

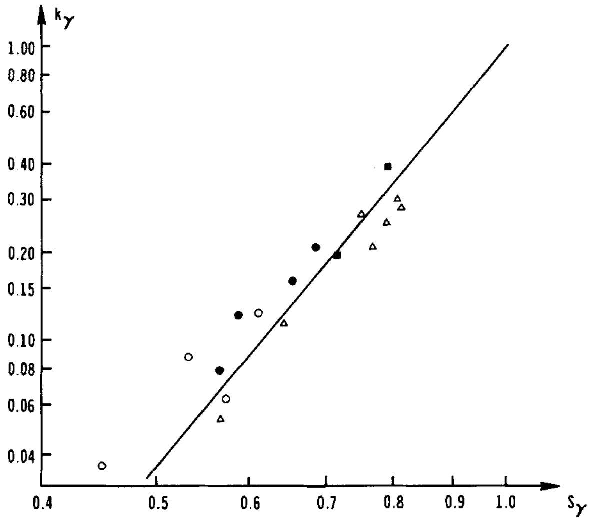

Figure 5. Gas relative permeability: logarithmic scale.

Figures 5 and 6 show how Eq. 48 compares to experimental data. The experimental values of k_γ for each point were found by combining Eqs. 41 and 47 to get

$$k_\gamma = \frac{1}{\psi_\gamma}\left(A\frac{Re_\gamma^*}{Ga_\gamma^*} + B\frac{Re_\gamma^{*2}}{Ga_\gamma^*}\right) \tag{49}$$

The scatter of the data is larger than for the liquid relative permeability correlation. However, the data seem to follow the same general trend independently of the type of packing used. It is important to notice that all the data analyzed correspond to the air-water system.

The high value of the exponent in Eq. 48 makes the equation very sensitive to the value of S_γ so that small changes in S_γ induce larger changes in k_γ than in k_β. This fact accounts for part of the increased scatter.

We can now combine Eqs. 41, 42 and 48 to get a general correlation for predicting the dimensionless pressure drop in gas-liquid cocurrent downflow through packed beds,

$$\psi_\gamma = \frac{1}{S_\gamma^{4.8}}\left\{A\frac{Re_\gamma^*}{Ga_\gamma^*} + B\frac{Re_\gamma^{*2}}{Ga_\gamma^*}\right\} \tag{50}$$

Equation 50 gives ψ_γ as a function of the operating conditions, the Ergun coefficients and S_γ. The value of S_γ can be predicted by means of Eqs. 21, 31, 32, 47 and 48, which combined yield

$$\frac{1}{\delta_\beta^{2.43}}\left\{A\frac{Re_\beta^*}{Ga_\beta^*} + B\frac{Re_\beta^{*2}}{Ga_\beta^*}\right\} - \frac{\rho_\gamma}{\rho_\beta}\frac{1}{S_\gamma^{4.8}}\left\{A\frac{Re_\gamma^*}{Ga_\gamma^*} + B\frac{Re_\gamma^{*2}}{Ga_\gamma^*}\right\} = 1 \tag{51}$$

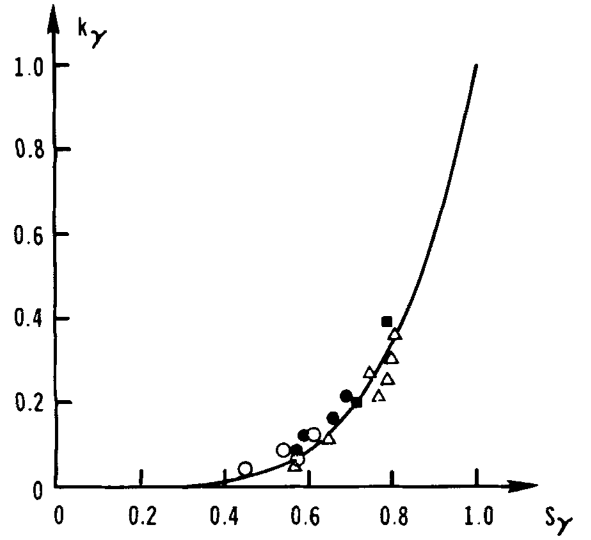

Figure 6. Gas relative permeability: linear scale.

TABLE 7. MEAN RELATIVE DEVIATIONS: PRESSURE DROP AND LIQUID HOLDUP PREDICTION

d_c/d_p	n_ϵ	$\hat{e}_\epsilon(\%)$	n_ψ	$\hat{e}_\psi(\%)$
>20	17	14.2	17	25.7
<20	42	12.2	32	19.9
All Data	59	12.7	49	21.9

In the equation above, we have considered $\rho_\beta \gg \rho_\gamma$.

Equation 51 relates the liquid holdup to the operating conditions. The liquid holdup appears in the definitions of the reduced liquid saturation

$$\delta_\beta = \frac{\epsilon_\beta - \epsilon_\beta^o}{\epsilon - \epsilon_\beta^o} \tag{24}$$

and the gas saturation

$$S_\gamma = 1 - \frac{\epsilon_\beta}{\epsilon} \tag{52}$$

Given a set of operating conditions and physical properties of the fluids, ϵ_β and, therefore, δ_β and S_γ, can be calculated by means of Eq. 51 and then ψ_γ is predicted by applying Eq. 50.

This procedure was performed for *all* the data collected from the literature and listed in Table 6. Table 7 shows the mean relative deviations for all the data analyzed, according to the influence of the ratio d_c/d_p. In this table n represents the number of data points.

We cannot draw conclusions from the results in Table 7 regarding the effect of d_c/d_p since the mean deviations for the case $d_c/d_p < 20$ are very close to those corresponding to $d_c/d_p > 20$. The fact that $\hat{e}_\epsilon$ and $\hat{e}_\psi$ actually decrease may be due to the small number of experimental points analyzed. Furthermore, the ratio of column to particle diameter was always greater than 10 so that the wall effect was probably of limited importance.

We see that the correlations proposed give good agreement overall (12.7% for ϵ_β and 21.9% for ψ_γ) in spite of the high sensitivity of ψ_γ on the predicted value of ϵ_β.

Figures 7 and 8 show how well the correlations compare with experimental data. Note that the majority of the pressure drop data lies in the ±30% region (Figure 7), whereas the liquid holdup data lie between the −20% and the +20% lines (Figure 8).

The next step is to compare the correlations developed with other correlations in the literature. For the pressure drop we compared our correlation with those developed by Larkins (1959) and Midoux et al. (1976). These were the more general correlations with respect to the amount of data studied. Larkins' correlation is recommended by Shah (1979) to predict pressure drops in gas-liquid cocurrent downflow in packed beds.

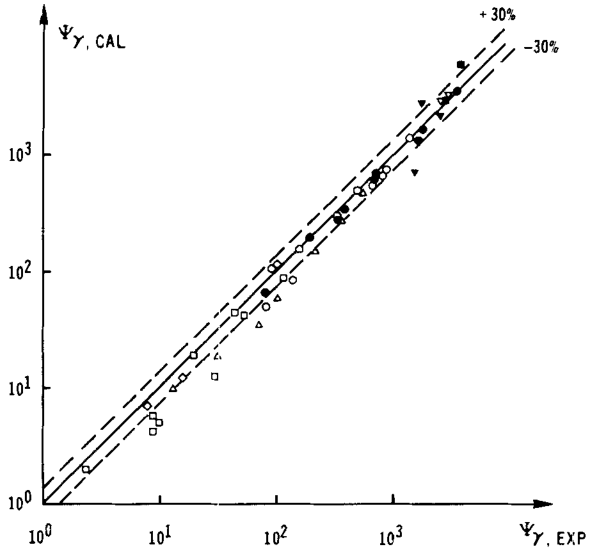

Figure 7. Comparison with experimental data: pressure drop.

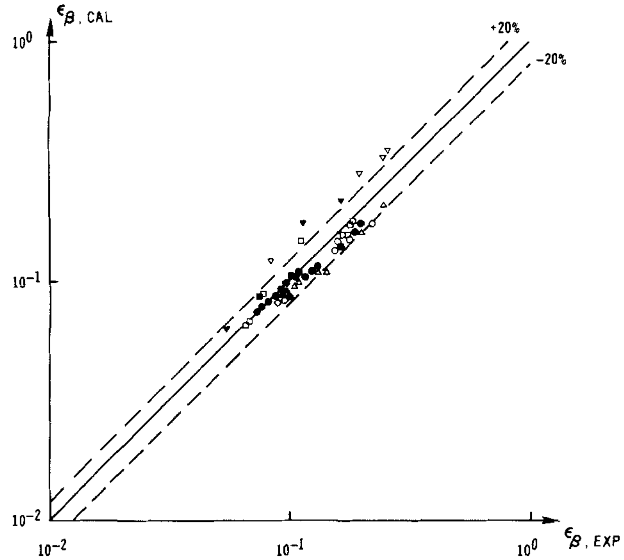

Figure 8. Comparison with experimental data: liquid holdup.

Table 8 shows the comparison between correlations for the 49 experimental pressure drops. Both correlations give a very large mean deviation due to the fact that they overestimate ψ_γ for low pressure drops. The correlation of Midoux et al. (1976) was developed from data for cylinders and spheres. If we take only these geometries, the mean deviation drops from 101.5 to 34.0% (n_ψ = 24), which is a more acceptable value. This fact suggests that the Midoux et al. correlation should be applied only for those geometries, which restricts the generality of the correlation.

The liquid holdup predictions are compared to the correlations of Midoux et al. (1976) and Clements and Schmidt (1980). The results are presented in Table 9. The correlation by Midoux et al. gives very poor results. The predictions of Clements and Schmidt's correlation are acceptable and they are much better when applied to experimental points in the range of operating conditions that they analyzed in their work. The correlation developed in this work gives the lowest mean relative deviation.

SUMMARY AND RECOMMENDATIONS

In this section we present a summary of the correlations developed in this work along with the ranges of independent variables analyzed.

Most of the data used in estimating parameters corresponded to the air-water system so that these correlations should be applied to other systems only as an approximation.

Prediction of Static Holdup

$$\epsilon^o_\beta = \frac{1}{20 + 0.9E\ddot{o}^*}$$

$$0.25 \leq E\ddot{o}^* \leq 30$$

Mean absolute deviation: 24.8% in 17 experimental points. The sources of experimental data are listed in Table 1.

TABLE 8. MEAN RELATIVE DEVIATIONS: PRESSURE DROP PREDICTION

Correlation	$\hat{e}_\psi(\%)$
Larkins (1959)	83.0
Midoux et al. (1976)	101.5
This Work	21.9

TABLE 9. MEAN RELATIVE DEVIATIONS: LIQUID HOLDUP PREDICTIONS

Correlation	$\hat{e}_\epsilon(\%)$
Midoux et al. (1976)	51.8
Clements and Schmidt (1980)	31.4
This Work	12.7

Prediction of Liquid Holdup for Stagnant Gas Phase

$$\delta_\beta = \left(A\frac{Re^*_\beta}{Ga^*_\beta} + B\frac{Re^{*2}_\beta}{Ga^*_\beta}\right)^{0.41}$$

$$0.2 \leq Re^*_\beta \leq 2{,}000$$

$$3{,}000 \leq Ga^*_\beta \leq 4 \times 10^8$$

Mean absolute deviation: 15.2% in 98 experimental points. The sources of experimental data are listed in Table 2.

Prediction of Liquid Holdup for Gas-Liquid Cocurrent Down-Flow

$$\frac{1}{\delta^{2.43}_\beta}\left\{A\frac{Re^*_\beta}{Ga^*_\beta} + B\frac{Re^{*2}_\beta}{Ga^*_\beta}\right\} - \frac{\rho_\gamma}{\rho_\beta}\frac{1}{S^{4.8}_\gamma}\left\{A\frac{Re^*_\gamma}{Ga^*_\gamma} + B\frac{Re^{*2}_\gamma}{Ga^*_\gamma}\right\} = 1$$

If data for single-phase flow are not available, use

$$A = 180$$

$$B = 1.8$$

$$5 \leq Re^*_\gamma \leq 6{,}000$$

$$5 \leq Re^*_\beta \leq 1{,}500$$

$$150 \leq Ga^*_\gamma \leq 5 \times 10^5$$

$$4 \times 10^4 \leq Ga^*_\beta \leq 1.3 \times 10^8$$

Gas-liquid continuous regime, with a mean absolute deviation of 12.7% in 59 experimental points.
The sources of experimental data are listed in Table 6.

Prediction of Pressure Drop

$$\psi_\gamma = \frac{1}{S^{4.8}_\gamma}\left\{A\frac{Re^*_\gamma}{Ga^*_\gamma} + B\frac{Re^{*2}_\gamma}{Ga^*_\gamma}\right\}$$

If the liquid holdup is unknown, it should be predicted first by using the correlation for the case of gas-liquid cocurrent downflow.

Range of variables: same as for liquid holdup.

Mean absolute deviation: 21.9% in 49 experimental points. The sources of experimental data are listed in Table 6.

It is interesting to point out that the correlations for predicting liquid holdup and pressure drop were found by calculating only *two* adjustable parameters (the exponents in the relations between k_β and δ_β, and k_γ and S_γ) and determining the equality of viscous and inertial relative permeabilities.

ACKNOWLEDGMENTS

The authors wish to thank FONINVES of Venezuela for the financial support of one of the investigators (A. E. Sáez). This work was also partially funded by the Thermal Sciences Div. of Sandia National Laboratory in Livermore, CA, and we gratefully acknowledge their contribution to this effort.

NOTATION

A = constant in the viscous term of the Ergun equation
B = constant in the inertial term of the Ergun equation
d_e = packing equivalent diameter
d_p = particle nominal diameter

$\hat{e}$ = mean relative deviation.
$E\ddot{o}$ = Eötvös number, defined by Eq. 4
$E\ddot{o}^*$ = Eötvös number based on the hydraulic diameter, defined by Eq. 7
F = interfacial force
g = acceleration of gravity
Ga^* = Galileo number, $\rho^2 g d_e{}^3 \epsilon^3/\mu^2(1-\epsilon)^3$
J = Leverett J function
K = permeability of the porous medium
k_α = relative permeability of the α phase, viscous regime
$k_{\alpha i}$ = relative permeability of the α phase, inertial regime
l = characteristic length of the local scale
L = liquid phase mass flow rate per unit of column area
$\mathcal{L}$ = length over which $\Delta\langle\mathcal{P}\rangle$ is measured
P = absolute pressure
$\mathcal{P}$ = absolute pressure including gravitational contributions
P_c = capillary pressure
Re^* = Reynolds number, $\rho\langle v\rangle d_e/(1-\epsilon)$
S_α = saturation of the α phase, ϵ_α/ϵ
S_p = external area of a packing particle
v = velocity
V_p = volume of a packing particle

Greek Letters

δ = reduced saturation
ϵ = porosity
ϵ_α = volume fraction of the bed occupied by the α phase
ϵ_α^o = residual volume fraction of the bed occupied by the α phase
ϵ_α^D = dynamic holdup of α phase, $\epsilon_\alpha - \epsilon_\alpha^o$
θ_c = contact angle
μ = viscosity
ρ = density
σ = surface tension
ψ = dimensionless pressure drop, defined by Eq. 10

Subscripts

i = number of Tergitol solution
o = referred to water
β = liquid phase
γ = gas phase
ϵ = referred to liquid holdup
ψ = referred to pressure drop

Other Symbols

$\langle\ \rangle$ = phase average, $\langle v_\alpha\rangle$ is the superficial velocity of the α phase
$\langle\ \rangle^\alpha$ = intrinsic phase average, $\langle\ \rangle/\epsilon_\alpha$

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Manuscript received March 18, 1983; revision received October 18, and accepted October 20, 1983.

Development of Improved Bubble Disruption and Dispersion Technique by an Applied Electric Field Method

With the aid of nonuniform DC electric fields, an improved bubble disruption and dispersion technique has been developed for dielectric fluid bubble column reactors. The basic mechanisms which lead to bubble disruption and dispersion in the system have been studied both experimentally and theoretically. The influence of operating variables, liquid properties and electrode geometry on the present applied electric field method has been conducted. The efficiency of power consumption by using the present technique has also been revealed.

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SCOPE

It is well known that bubble column reactors have a wide range of applications in many components of chemical industrial process such as absorption, catalytic slurry reactions, bioreactions, coal liquefaction, etc. These reactors are preferred because of simplicity of operation and low operating costs. Bubble column reactors have recently been reviewed by Mashelkan (1970) and Shah et al. (1982). Main objectives of designing better mass transfer bubble column reactors, other than energy efficiency, are to have better insight of the residence time, the uniformity, and the gas-liquid interfacial area.

In this paper, a new type of bubble dispersion technique based on applied electric field method for a dielectric fluid is introduced, and the basic bubble disruption and dispersion mechanisms as well as system efficiency are investigated both theoretically and experimentally. To understand the uniformity, interfacial area, and residence time, the analysis started with mechanism of bubble disruption and dispersion, and where this process would occur inside the reactors. The numerical analyses of electric field distribution have been conducted with the consideration of existence of bubbles, and compared with experiments at bubble injection point. The influence of operational variables, liquid properties and electrode geometry on the system efficiency has been investigated experimentally and compared with the existing techniques.

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